

4/12/18

"Yesterday is history, tomorrow a mystery, today is a gift called the present."-Unknown

HW: "Exponential Modeling Day 1" homework section #1-8

AIM: How do we use logs?

Warm Up:

Exponential functions are very important in modeling a variety of real world phenomena because certain things either increase or decrease by **fixed percentages** over given units of time. You looked at this in Common Core Algebra I and in this lesson we will review much of what you saw.

Exercise #1: Suppose that you deposit money into a savings account that receives 5% interest per year on the amount of money that is in the account for that year. Assume that you deposit \$400 into the account initially.

- (a) How much will the savings account increase by over the course of the year?

$$5\% = .05$$

$$400(.05) = 20$$

- (b) How much money is in the account at the end of the year?

$$400 + 20 = \$420$$

- (c) By what single number could you have multiplied the \$400 by in order to calculate your answer in part (b)?

$$1.05$$

$$400 + 400(.05) = 420$$

$$400(1 + .05) = 420$$

$$400(1.05) = 420$$

- (e) Give an equation for the amount in the savings account $S(t)$ as a function of the number of years since the \$400 was invested.

$$S(t) = 400(1.05)^t$$

- (d) Using your answer from part (c), determine the amount of money in the account after 2 and 10 years. Round all answers to the nearest cent when needed.

$$400(1.05)(1.05) = \$441$$

$$400(1.05)^2 = 441$$

$$400(1.05)^{10} = \$651.56$$

- (f) ~~Using a table or calculator~~ determine, to the nearest year, how long it will take for the initial investment of \$400 to double. Provide evidence to support your answer.

$$800$$

$$\frac{800}{400} = \frac{400(1.05)^t}{400}$$

$$2 = 1.05^t$$

⊗ Rewrite in log form

$$t = \log_{1.05} 2$$

$$t \approx 14 \text{ years}$$

The thinking process from *Exercise #1* can be generalized to any situation where a quantity is increased by a fixed percentage over a fixed interval of time. This pattern is summarized below:

INCREASING EXPONENTIAL MODELS

If quantity A is known to increase by a fixed percentage rate r , in decimal form, then A can be modeled by

$$A(t) = P(1 + r)^t$$

where P represents the initial amount of present at $t = 0$ and t represents time.

Growth/Decay

t and r need to be same units

$A(t)$ = Final Amount

P = Starting Amount

r = rate

t = time

$$A(t) = P(1 \pm r)^t$$

(+) for growth

(-) for decay

Exercise #2: Which of the following gives the savings S in an account if \$250 was invested at an interest rate of 3% per year?

(1) $S = 250(4)^t$

(3) $S = (1.03)^t + 250$

(2) $S = 250(1.03)^t$

(4) $S = 250(1.3)^t$

↑
increasing

$$S = P(1 + r)^t$$

$$S = 250(1 + .03)^t$$

$$= 250(1.03)^t$$

Decreasing exponentials are developed in the same way, but have the percent subtracted, rather than added, to the base of 100%. Just remember, you are ultimately multiplying by the percent of the original that you will have after the time period elapses.

Exercise #3: State the multiplier (base) you would need to multiply by in order to decrease a quantity by the given percent listed.

(a) 10%

$$A = P(1 - .10)^t$$

$$A = P(.9)^t$$

$$\boxed{.90}$$

(b) 2% = .02

$$\boxed{.98}$$

(c) 25%

$$\boxed{.75}$$

(d) 0.5% = .005

$$\boxed{.995}$$

DECREASING EXPONENTIAL MODELS

If quantity A is known to decrease by a fixed percentage rate r , in decimal form, then A can be modeled by

$$A(t) = P(1 - r)^t$$

where P represents the initial amount of present at $t = 0$ and t represents time.

Exercise #4: If the population of a town is decreasing by 4% per year and started with 12,500 residents, which of the following is its projected population in 10 years? Show the exponential model you use to solve this problem.

(1) 9,230

(3) 18,503

(2) 76

(4) 8,310

$$A(t) = P(1 \pm r)^t$$

$$A(t) = 12500(1 - .04)^{10}$$

$$A(t) = 12500(.96)^{10}$$

Exercise #5: The stock price of WindpowerInc is increasing at a rate of 4% per week. Its initial value was \$20 per share. On the other hand, the stock price in GerbilEnergy is crashing (losing value) at a rate of 11% per week. If its price was \$120 per share when Windpower was at \$20, after how many weeks will the stock prices be the same? Model both stock prices using exponential functions. Then, find when the stock prices will be equal graphically. Draw a well labeled graph to justify your solution.

Windpower Inc.

$$A = 20(1 + .04)^t$$

$$A = 20(1.04)^t$$

Gerbil Energy

$$A = 120(1 - .11)^t$$

$$A = 120(.89)^t$$

$$t = 11.5 \text{ weeks}$$